

## INTERACTING HOT DARK MATTER

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We discuss the viability of a light particle ( $\sim 30$  eV neutrino) with strong self-interactions as a dark matter candidate. The interaction prevents the neutrinos from free-streaming during the radiation dominated regime so galaxy sized density perturbations can survive. Smaller scale perturbations are damped due to neutrino diffusion. We calculate the power spectrum in the imperfect fluid approximation, and show that it is damped at the length scale one would estimate due to neutrino diffusion. The strength of the neutrino-neutrino coupling is only weakly constrained by observations, and could be chosen by fitting the power spectrum to the observed amplitude of matter density perturbations. The main shortcoming of our model is that interacting neutrinos can not provide the dark matter in dwarf galaxies.

# 1 introduction

Observations on many scales imply that most of the matter in the universe is not emitting electromagnetic radiation [1]. The ratio of the energy density in this unseen matter component (usually referred as dark matter or DM) to the critical density today, is defined as  $\Omega_o$  and is under debate. Various dynamical determinations of  $\Omega_o$  on the scales of galaxies and clusters give  $\Omega_o \simeq .2 \pm .1$  [1]. Observations of large velocity flows suggest that  $\Omega_o$  could be larger than this, possibly  $\Omega_o \simeq 1$  with large uncertainties [2, 3]. Theoretical prejudice favours  $\Omega_o = 1$ , because this is more natural than  $\Omega_o < 1$ , and because it is predicted by inflation. However, in this case, one needs  $\Omega \sim .8$  smooth on galaxy scales.

Primordial nucleosynthesis implies that the energy density in baryons satisfies  $.003 < \Omega_B < .06$  [4], which might just be consistent with the observations of  $\Omega_o$ . However, it is very difficult to build models involving only baryons that produce the large scale structure and the Cosmic Microwave Background (CMB) temperature fluctuations we see today [5, 6], so it is usually assumed that the dark matter is made of something other than baryons. The list of potential candidates is long; the popular possibilities are massive neutrinos, axions, and supersymmetric particles.

In the standard picture, the dark matter (DM) is in thermal equilibrium with the rest of the Universe at some early epoch, but effectively non-interacting when length scales of interest for structure formation come into the horizon. In this case, the only free parameter is the mass of the DM particle. If it had  $m \gtrsim 1$  GeV, it would be non relativistic at all times of interest for structure formation, and it would behave like a pressureless fluid. If it was light, it would be relativistic until cluster scales came within the horizon, and perturbations on smaller scales would be damped by free-streaming. As a result, in this second type of model (called hot dark matter or HDM models) clusters are produced first and must fragment into galaxies. This “top–down” galaxy formation scenario is difficult to reconcile with observations [7] which indicate that galaxies and small scale structure formed first. A model where the dark particles behave non–relativistically is therefore preferred. The standard cold dark matter (CDM) model was found to have too much power on small scales when normalized to the amplitude of the measured COBE/DMR temperature anisotropies and several modifications were introduced. Another possibility is a “warm” dark matter particle with a mass ( $\sim$  keV) intermediate between that of “hot” ( $\sim$  eV) and “cold” ( $>$  GeV) [9]. Currently, models with a mixture of hot and cold dark matter (MDM) are favoured [8].

It is usually assumed that during structure formation ( $T_\gamma < 100$  eV) the evolution of the Universe was controlled by electromagnetism and gravitation, while new particle physics takes place at accelerator energies. However, it is possible that the dark matter particles could have some unexpected interaction during the epoch of structure formation, despite this occurring at a low energy scale. Some examples of this have been previously studied [10, 11, 12, 13]. In this paper, we consider a simple interacting dark matter model; we assume that the dark matter consists of light particles with a large cross section for scattering off each other (we discuss later the meaning of “large”). For definiteness, we take the light particle to be a neutrino with mass  $\sim 20 - 30$  eV (as for HDM). Our interest is to determine whether, in the presence of hot dark matter self-interactions, density perturbations on scales smaller than galaxy clusters remain undamped. In normal HDM, they are washed out by free streaming of relativistic particles; in an interacting model, one would rather expect the perturbations to be damped on a smaller scale by diffusion (Silk damping). We will show that this is in fact the case, and compute on what scale they are damped.

The question of whether “sticky neutrinos behave like cold dark matter” has been previously addressed by Raffelt and Silk [10]. They argue that light ( $m \sim 20$  eV) interacting neutrinos will dissipate small scale perturbations ( $<$  galaxy sized) by “Silk damping”, but there will be no free streaming. This result was briefly questioned by Dicus *et al.* in their paper about neutrino interactions in supernovae [14], where they suggest that cosmological density perturbations in a neutrino fluid would dissipate at the speed of sound <sup>1</sup>. It is well known that perturbations in a relativistic perfect fluid in a radiation dominated Universe neither grow nor are damped [15]. However, if the mean free path of the neutrinos is non-zero, they do not constitute a perfect fluid. There are contributions to the fluid viscosity and heat conduction coefficients that are proportional to the mean free path. The perturbations in the relativistic neutrino fluid in the early Universe will be damped by the fluid imperfections, and as Weinberg [16] showed for Silk damping, the damping length scale can be estimated by the random walk argument used in [10]. Raffelt and Silk’s result is therefore a correction to a perfect fluid analysis, not a contradiction to it.

The aim of this paper is to study “sticky neutrinos” as dark matter in the imperfect fluid approximation. Providing that the particle mean free path is much shorter than the lengthscale of the density perturbations, this is a sensible approximation because it takes into account the non-zero mean free path of the particles, but avoids the complexity of the Boltzmann equation. In section 2, we briefly review the experimental bounds on the  $\nu\nu$  scattering cross section, and discuss theories in which such interactions could be strong. In the first part of the third section, we review the Boltzmann equation and its relation to the fluid approximation, in the interest of making this paper more self-contained. In the second part of this section, we estimate the scale on which perturbations in an “sticky hot dark matter” Universe would be damped. We do this by treating the neutrinos as a perfect fluid in an expanding Friedmann-Robertson-Walker Universe to lowest order, and then computing the viscosity as a perturbation proportional to the neutrino mean free path. We briefly discuss the subsequent behaviour of the sticky neutrinos in galaxy formation, and calculate the power spectrum. In section 4 we show that interacting neutrinos cannot constitute the halo of dwarf galaxies. However, we argue that this is a potential problem not only for light fermions but for all dissipationless DM. Finally, we present our conclusions.

## 2 particle physics

Despite the precision to which most Standard Model quantities are known, there are no experimental bounds on the  $\nu\nu$  or  $\nu\bar{\nu}$  scattering cross section. This is hardly surprising, as it would be difficult to scatter neutrino beams off each other with a high enough luminosity to see anything within the Standard Model. The best constraint on the  $\nu\nu$  and  $\nu\bar{\nu}$  cross section comes from the observation of the neutrinos from SN1987A. These had to pass through the “cosmic neutrino background radiation” between here and the Large Magellanic Cloud (LMC) before arriving at the earth. Approximately the right number of neutrinos were detected, which places an upper bound on the  $\nu\nu$  interaction. This constraint has been computed in [17]; we briefly outline their arguments here.

The “cosmic background” of light stable neutrinos has a number density  $n_\nu = \frac{7}{8}(T_\nu/T_\gamma)^3 n_\gamma$ , where  $T_\nu/T_\gamma$  depends on how many different families of particles annihilate into photons after

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<sup>1</sup>They note, however, that their flat space results are not directly applicable to cosmology.

the neutrino gas decouples from the photons. In this paper, we will assume  $(T_\nu/T_\gamma)^3 = 4/11$ ,  $n_\nu \simeq n_\gamma/3$ .

The flux of  $E_\nu \sim 10$  MeV neutrinos from the supernova will arrive at the earth if the mean free path  $\lambda$  of a supernova neutrino is of order the distance  $D \sim 2 \times 10^{23}$  cm from here to the LMC. The mean free path is computed in [17] with some care, but can be roughly estimated as

$$\lambda^{-1} \simeq \sigma n_\nu \quad (1)$$

where  $\sigma$  is the cross section for a neutrino to scatter off a  $\nu$  or a  $\bar{\nu}$ . Requiring  $\lambda/D \gtrsim 1$  gives

$$\sigma \lesssim 90 \text{ GeV}^{-2}, \text{ at } \sqrt{s_{sn}} = 20 \text{ keV} \quad (2)$$

where  $\sqrt{s_{sn}}$  is the centre-of-mass energy of the collision. Since we assume that the neutrinos are the dark matter, they have masses of order 20 — 30 eV, and  $s_{sn} \simeq 2E_\nu m_\nu \simeq (20 \text{ keV})^2$ .

We are interested in the neutrino self interaction cross section at centre-of-mass energy scales near 10 eV. This is because galaxy scales come into the horizon when  $T \gtrsim 10$  eV, and the neutrinos become non-relativistic shortly thereafter; if galaxy scale perturbations are going to be damped, it will be during this period. We therefore need to scale (2) down to lower values of  $s$ . From a phenomenological point of view,  $\sigma$  can scale as  $\sigma(s) = \sigma(s_{sn})s/s_{sn}$  (due to exchange of a boson with  $m \gg s$ ), or as  $\sigma(s) = \sigma(s_{sn})s_{sn}/s$  (exchange of a boson with  $m \ll s$ ). Galaxy-sized perturbations will be undamped in both cases. We will concentrate on a model where the cross section scales as  $s$ . The other possibility, that it would scale as  $1/s$ , would lead to an interaction strong enough to affect the dynamics of groups of galaxies today. Long range dark matter interactions have been considered in [13].

Sticky neutrino models are not easy to construct. The obstacle is that the Standard Model neutrinos are members of the same  $SU(2)$  doublet as the electrons, so it is difficult to give them strong self-interactions without contradicting present experimental data. This can be avoided by introducing right-handed gauge singlet neutrinos [18], but then one has difficulties with the primordial nucleosynthesis bound on the number of light degrees of freedom present at  $T \sim 1$  MeV in the early Universe [4]:  $N_\nu \lesssim 4$ . Attempting to construct a neutrino mass matrix that is consistent with this constraint, and various indications of neutrino oscillations (solar neutrino problem, atmospheric neutrino deficit,...) requires more fine-tuning.

Sticky neutrinos were originally studied in the context of a modified triplet majoron model [19, 20], where the majoron (the nambu-goldstone boson associated with the spontaneous breaking of lepton number  $L$ ) was more massive than the lightest neutrino. The neutrinos interact via the exchange of the majoron. However, the triplet majoron couples to the  $Z$ , and has not been detected at LEP, so this model is ruled out. If one extends the Standard Model fermion content to include right-handed neutrinos, then these can acquire majorana masses by coupling to a singlet scalar with a vacuum expectation value, and Dirac masses with the left-handed neutrinos by coupling to the Higgs. This is the singlet majoron model [21], and is experimentally only weakly constrained. The angular component of the scalar is the majoron, and its coupling to the light neutrinos is proportional to the neutrino mass. This model can be fine-tuned to produce interacting light neutrinos. The right-handed neutrino majorana masses must be of order the Dirac masses, to allow a big enough neutrino scattering cross section by majoron exchange. To avoid having too many neutrinos at nucleosynthesis, one can make one family light enough that the right handed component is not yet in equilibrium, and another heavy enough to decay beforehand. (We have added strong neutrino self-interactions, so it is possible to arrange fast decays for a heavy neutrino). This adds up to three neutrinos plus the

majoron ( $= .6\nu$ ) at nucleosynthesis. One must then impose flavour symmetries, to prevent the middle neutrino, who is the dark matter candidate, from decaying. This mass matrix can not explain the solar neutrino problem by oscillations.

Alternatively, we can disconnect the exchanged scalar from the neutrino mass generation mechanism. In this way, we can get “three neutrinos at nucleosynthesis”. If we introduce a few MeV scalar with gauge strength couplings to the dark matter neutrino, it induces strong enough interactions to preserve galaxy scale perturbations, and may not be present as a relativistic degree of freedom at nucleosynthesis. We must again arrange the neutrino mass matrix so that the heaviest neutrino decays before nucleosynthesis while the middle one, who is our dark matter candidate, is stable, and the lightest one’s right-handed component is not in thermal equilibrium at  $T \sim \text{MeV}$ . The light neutrino must therefore have a very small Dirac mass, and very weak coupling to the scalar. There is again no possibility of explaining the solar neutrino problem.

We can construct a neutrino mass matrix that could explain the solar neutrino problem if we allow four neutrinos at nucleosynthesis. We introduce a  $\sim$  few MeV scalar whose interactions are unrelated to the neutrino masses, give it gauge strength couplings to the 30 eV neutrino, and very weak interactions with the other two neutrinos, who are lighter. We assume that these other two neutrinos also have very small Dirac masses, so their right-handed components are not in thermal equilibrium at nucleosynthesis. This makes four neutrinos at nucleosynthesis. We can then allow the left-handed components of the two lighter neutrinos to have a majorana mass difference that explains the solar neutrino deficit.

Another possibility would be to disconnect the interacting hot dark matter from the neutrinos, and imagine that there is a light real self-interacting scalar as well as three “standard” neutrinos. This would give 3.6 neutrino families at nucleosynthesis, and the neutrino mass matrix is still free, but one loses the great advantage of neutrinos over other dark matter candidates, which is that they are known to exist. In any case, we are mainly interested in interacting hot dark matter from a phenomenological point of view, and do not advocate any particular model.

### 3 Evolution of density perturbations.

The aim of this section is to discuss the formalism for calculating the evolution of linear perturbations in interacting dark matter, and apply it to the case of “sticky neutrinos”. In the first part, we review the Boltzmann equation and its relationship to the fluid approximation. We discuss how to compute the viscosity  $\eta$ , and the heat conduction  $\chi$  for a slightly imperfect fluid, and determine their contribution to the damping of perturbations. In the second part, we calculate  $\eta$  for a neutrino fluid, and show that the estimate of the damping scale made by Raffelt and Silk [10] is essentially correct.

We use the metric

$$ds^2 = dt^2 - a^2(t)[\delta_{ij} + h_{ij}(\vec{x}, t)]dx^i dx^j . \quad (3)$$

where  $h_{ij}$  is the metric perturbation in synchronous gauge. We are interested in the effect of particle interactions on the evolution of matter density perturbations within the horizon. On those scales, metric perturbations are negligible compared to pressure gradients, damping and other hydrodynamical effects [22, 23]. We therefore neglect the metric perturbations in our analytic estimates, although they are included in our numerical calculation of the power spectrum.

We define  $k$  to be the magnitude of the comoving three-momentum, and  $\gamma_i$  its directional cosine: if  $|p_i p^i|^{1/2} = p$  is the magnitude of the physical momentum,  $p = k/a$ ,  $p_i = -k\gamma_i$ , and  $p^i = a^{-1}p\gamma_i$ . Roman indices run from 1 to 3, and greek from 0 to 3.

### 3.1 The Boltzmann equation and the fluid approximation.

The Boltzmann equation describes the evolution of the phase space density of the particles  $f(x^i, p_i, t)$ , and provides a framework in which one can calculate the growth and survival of density perturbations once they come within the horizon. It can be written as

$$\mathcal{L}[f] = C[f] \quad (4)$$

where  $\mathcal{L}[f]$  is the derivative of  $f$  along the path in phase space of the particles, and  $C[f]$  is the collision integral. If the particles described by  $f$  have no collisions,  $f$  is constant along a particle's path in phase space, or  $\mathcal{L}[f] = 0$ . In a relativistic fluid under the influence of gravitation,  $\mathcal{L}[f]$  is defined as [24]

$$\mathcal{L}[f] = p^\mu \frac{\partial f}{\partial x^\mu} - \Gamma_{\nu\beta}^\mu p^\nu p^\beta \frac{\partial f}{\partial p^\mu} . \quad (5)$$

$C[f]$  parametrizes the changes in the distribution function due to the particle collisions. The integral representing the change in the distribution function  $f_a$  for particle  $a$  due to the process  $a + b \rightarrow c + d$  is [24]

$$C[f_a] = \frac{1}{2} \int d\Pi_b d\Pi_c d\Pi_d \hat{\delta}^4 |\mathcal{M}_{ab \rightarrow cd}|^2 (f_a f_b - f_c f_d) \quad (6)$$

where  $|\mathcal{M}|^2$  is the matrix element squared for the process  $a + b \rightarrow c + d$ , including spin sums and averages, and

$$d\Pi = \frac{d^3 p}{2E(2\pi)^3} \quad (7)$$

$$\hat{\delta}^4 = (2\pi)^4 \delta^4(p_a + p_b - p_c - p_d) . \quad (8)$$

Implicitly, in equation (6) we have used Maxwell-Boltzmann statistics by neglecting the occupation in the final phase space. This approximation is justified if the chemical potentials of all particles are negligible at the temperature scales involved. But even in this limit, the solution of the Boltzmann equation is complicated.

One can avoid dealing with the collision term by integrating (4) over quantities that are conserved in the interaction described by  $C[f]$  [25]. This eliminates the right-hand-side of (4). For instance, if  $p_\mu$  is the four-momentum of the particle  $a$ , one can write

$$\int d\Pi_a p_a^\mu \mathcal{L}[f_a] = \int d\Pi_a p_a^\mu C[f_a] \quad (9)$$

and it is easy to show from symmetry considerations that the integrand on the right hand side is proportional to  $p_a^\mu + p_b^\mu - p_c^\mu - p_d^\mu$ , so the integral is zero. Therefore, one obtains

$$\int d\Pi_a p_a^\mu \mathcal{L}[f_a] = 0 . \quad (10)$$

In the case of sticky neutrinos, the scattering interaction conserves particle number, so we also have

$$\int d\Pi_a \mathcal{L}[f_a] = 0 . \quad (11)$$

Defining the stress-energy tensor for a single species as

$$T^{\mu\nu} = 2 \int d\Pi f p^\mu p^\nu , \quad (12)$$

and the number density vector

$$N^\mu = 2 \int d\Pi f p^\mu , \quad (13)$$

it is easy to check, in a Friedmann-Robertson-Walker Universe, that (10) and (11) are equivalent to the usual conservation equations

$$T^{\mu\nu}_{;\nu} = T^{\mu\nu}_{;\nu} + \Gamma^\mu_{\nu\kappa} T^{\nu\kappa} + \Gamma^\nu_{\nu\kappa} T^{\mu\kappa} = 0 \quad (14)$$

and

$$N^\mu_{;\mu} = 0 . \quad (15)$$

It is clear, however, that since these are only conservation equations, they do not contain enough information to specify the dynamics. We still need to solve the Boltzmann equation (4) to obtain  $f$ . Since the Universe was homogeneous and isotropic to better than one part in  $10^5$  when it was radiation dominated, one can solve the Boltzmann equation for perturbations away from the homogenous and isotropic background [15]. A second possibility, which can only be used for interacting particles, is to perturb away from a perfect fluid. We shall use this second approach for sticky neutrinos, since we assume they are strongly interacting. These two schemes are not identical, because in the first case the background is a homogeneous and isotropic equilibrium distribution  $f \sim \exp\{-E/T(t)\}$ , and the perturbation in  $f$  corresponds to a density perturbation. In the second case, one perturbs away from a locally thermal distribution  $f \sim \exp\{-E/T(x,t)\}$ , so density perturbations are already present in the background solution. It is a two step procedure to find the evolution of the density perturbations in the fluid approximation. We first estimate the magnitude of the perturbation away from local thermal equilibrium using the Boltzmann equation. Then we calculate the contributions of these perturbations to the stress-energy tensor, and solve for the evolution of the density perturbations from the conservation equation (14). The perturbations away from local thermal equilibrium introduce new terms in (14) which can affect the growth of the density perturbations.

A perfect fluid is locally in thermal equilibrium, so we can write the phase space density for particles constituting an imperfect fluid as

$$f = g^{(0)}(E, x^i, t) + g^{(1)}(E, p_i, x^i, t) \quad (16)$$

where

$$g^{(0)}(E, x^i, t) = \frac{g}{(2\pi)^3} \exp\{-[E - \mu(x, t)]/T(x, t)\} \quad (17)$$

is the local equilibrium distribution,  $g$  is the number of degrees of freedom of the particle, and  $E = U^\alpha p_\alpha$ , where  $U^\alpha$  is the four-velocity of a particular observer.  $g^{(1)}$  measures the departure of the distribution from exact local equilibrium.

If the particles involved in an interaction are in local thermal equilibrium, then the collision term in the Boltzmann equation is zero; however,  $g^{(0)}$  does not necessarily satisfy the left hand side of the Boltzmann equation. Assuming that  $g^{(1)} \ll g^{(0)}$ , we have

$$E \frac{\partial g_a^{(0)}}{\partial t} + \frac{p \gamma_i}{a} \frac{\partial g_a^{(0)}}{\partial x^i} + H p^2 \frac{\partial g_a^{(0)}}{\partial E} = \frac{1}{2} \int d\Pi_b d\Pi_c d\Pi_d \hat{\delta}^4 |\mathcal{M}_{ab \rightarrow cd}|^2 \quad (18)$$

$$\times (g_a^{(1)} g_b^{(0)} + g_a^{(0)} g_b^{(1)} - g_c^{(1)} g_d^{(0)} - g_c^{(0)} g_d^{(1)}) . \quad (19)$$

In principle, to correctly compute the deviations from perfect fluid behaviour, we still have to evaluate the collision integral. However, if we only wish to estimate viscosity and heat conduction coefficients, the right hand side can be taken [25] (to within a factor of  $\sim 4$ ) as its first term  $\sim E_a \Gamma g^{(1)}$ , where  $\Gamma$  is the thermally averaged interaction rate for the reaction described by  $C[f]$ . This gives

$$g^{(1)} \simeq \frac{\mathcal{L}[g^{(0)}]}{E_a \Gamma} . \quad (20)$$

To get the damping scale of the perturbations correctly, we would need a better approximation, as can be found, for instance, in [26]. A more accurate approach would require a numerical integration of the coupled Einstein and Boltzmann equations that describe the behaviour of metric perturbations and the density field of the different particles [27].

If the mean time between particle collisions is  $\tau = \Gamma^{-1}$ , and  $t$  is the timescale associated with the density perturbation in the fluid, (20) implies  $g^{(1)} \sim g^{(0)} \tau / t$ . So  $g^{(1)} \ll g^{(0)}$ , and the fluid approximation is consistent, provided that the size of the density perturbations in the early Universe is much longer than the mean free path of the particles. In this limit, the phase space density to first order in  $\tau$  is

$$f = g^{(0)} + \frac{\tau}{E} \mathcal{L}[g^{(0)}] \quad (21)$$

We have not yet defined  $\mu$  or  $T$ , or discussed in which reference frame we are measuring the energy  $E$ . In principle  $E = p^\alpha U_\alpha$ , so we must specify  $U_\alpha$ . We follow Landau and Lifschitz [28], and take  $U_\alpha$  to be the velocity associated with energy flux. (Note that this is not the choice made in [16], so we can not immediately compare to Weinberg's equations). This is natural in the approximation scheme we have used, because

$$T^{0i} = 2 \int d\Pi E p^i g^{(1)} = 2 \int d\Pi p^i \mathcal{L}[g^{(0)}] = O(\tau^2) \simeq 0 \quad (22)$$

where the last equality follows from (10). We define  $T$  such that the equilibrium expression for the energy density is equal to the energy density in the reference frame where there is no energy flux, and  $\mu$  such that the equilibrium expression for the number density is equal to the measured number density in the same reference frame. (In other words, the perturbations  $g^{(1)}$  will not contribute to  $\rho$  or  $n$  in the reference frame where  $T^{0i} = 0$ ).

The general form of the stress-energy tensor and the particle flux vector  $N^\mu$  for a slightly imperfect relativistic fluid can be determined by requiring the entropy to be non-decreasing [16, 28]. In the rest frame of  $U$ , the velocity associated with energy flux, they are [28]

$$a^2 T^{\mu\nu} = \begin{bmatrix} a^2 \rho & 0 & 0 & 0 \\ 0 & P - (\zeta - \frac{2}{3}\eta) \partial_\alpha U^\alpha & -\eta(\partial^1 U^2 + \partial^2 U^1) & -\eta(\partial^1 U^3 + \partial^3 U^1) \\ 0 & -\eta(\partial^1 U^2 + \partial^2 U^1) & P - (\zeta - \frac{2}{3}\eta) \partial_\alpha U^\alpha & -\eta(\partial^3 U^2 + \partial^2 U^3) \\ 0 & -\eta(\partial^3 U^1 + \partial^1 U^3) & -\eta(\partial^3 U^2 + \partial^2 U^3) & P - (\zeta - \frac{2}{3}\eta) \partial_\alpha U^\alpha \end{bmatrix} \quad (23)$$



and

$$N^\mu = (n, \frac{\chi}{a} \left( \frac{nT}{\rho + P} \right)^2 \partial^i \left( \frac{\mu}{T} \right)). \quad (24)$$

Formal expressions for the heat conduction coefficient  $\chi$  and the bulk and ordinary viscosities  $\zeta$  and  $\eta$  have been elegantly calculated in [29] for a generic relativistic fluid. However, their integrals are difficult to evaluate. We can simply approximate the fluid parameters by substituting (21) into (12) and (13), and comparing to the formulae for  $T^{\mu\nu}$  and  $N^\mu$ . To extract  $\chi$  from  $N^\mu$  one must use the constraint (22). We will do this for sticky neutrinos in the next subsection.

Once  $T^{\mu\nu}$  for an imperfect fluid has been calculated, the conservation equation (14) can be used to study the evolution of density perturbations on scales much longer than the particle interaction length. As noted in [16], perturbations in a fluid of relativistic particles are principally damped by viscosity. If we neglect the metric perturbations (reasonable for sub-horizon sized modes) and the heat conduction coefficient, and decompose the density perturbation in Fourier modes, (14) implies

$$\dot{\delta} = -\frac{4}{3}\theta \quad (25)$$

$$\dot{\theta} + \frac{\dot{a}}{a}\theta = p^2 \left( \frac{\delta}{4} - \frac{\eta}{\rho}\theta \right) \quad (26)$$

where  $\theta \equiv ik_j U^j$ ,  $\delta \equiv \delta\rho/\rho$ ,  $k$  is the comoving three-momentum and  $p = k/a$  is the physical three momentum. These are the equations for the longitudinal component ( $\vec{U} \parallel \vec{p}$ ) of the perturbation; the transverse component is rapidly damped [16]. One can see from (25) and (26) that the perturbation oscillates and damps at a rate

$$\Gamma \sim \frac{2p^2\eta}{3(\rho + P)} \quad (27)$$

due to the fluid viscosity  $\eta$ . Including heat conduction  $\chi$  and bulk viscosity  $\zeta$  gives [16]

$$\Gamma = \frac{p^2}{2(\rho + P)} \left[ \zeta + \frac{4}{3}\eta + A\chi \right] \quad (28)$$

where  $A$  is a function of various fluid parameters

$$A = \left( \frac{\partial\rho}{\partial T} \right)_n^{-1} \left[ \rho + P - 2T \left( \frac{\partial P}{\partial T} \right)_n + v_s^2 T \left( \frac{\partial\rho}{\partial T} \right)_n - \frac{n}{v_s^2} \left( \frac{\partial P}{\partial n} \right)_T \right]. \quad (29)$$

$v_s$  denotes the speed of sound. This function  $A$  goes to zero as particles become extremely relativistic.

### 3.2 Perturbation Damping for Sticky Neutrinos

The length scale on which neutrino diffusion damps density perturbations increases with the age of the Universe until the neutrinos become non-relativistic. We can estimate the comoving scale on which density perturbations will be damped by treating the neutrinos as relativistic until  $T \simeq m_\nu/3$ , and neglecting any further motion after this temperature. A similar approximation provides the ordinary neutrino free streaming scale to within a factor of three, and since we only know the fluid viscosity and heat conduction to within the same order, it should be adequate.

In the extreme relativistic limit, the bulk viscosity  $\zeta$  is zero, so we will neglect it. To compute the viscosity  $\eta$ , we note from [28] that in the reference frame where  $U = (1, 0, 0, 0)$

$$T^{12} = -\eta \left( \frac{\partial U^1}{\partial x_2} + \frac{\partial U^2}{\partial x_1} \right) . \quad (30)$$

Substituting (21) into (12), we find

$$T^{12} = \frac{\tau}{2a^2} \int p^2 dp d\Omega \gamma^1 \gamma^2 p^\mu \frac{\partial U^\alpha}{\partial x^\mu} \frac{\partial g^{(0)}}{\partial U^\alpha} + \dots \quad (31)$$

where the “...” indicate the terms which are zero when the angular integral is performed. With the identity

$$\int d\Omega \gamma^i \gamma^j \gamma^k \gamma^l = \frac{4\pi}{15} (\delta^{ij} \delta^{kl} + \delta^{ik} \delta^{jl} + \delta^{il} \delta^{kj}) \quad (32)$$

this gives in the relativistic limit

$$\eta = \frac{4\tau n T}{5} \quad (33)$$

which agrees with the result in [16, 30, 31].

The heat conduction coefficient  $\chi$  can be estimated by substituting (21) into (13), (with the constraint (22)) and comparing to (24). In the relativistic limit, we get

$$\chi = \frac{4\tau n}{3} \quad (34)$$

which is a factor of 3 smaller than the result in [16, 31] but is within the errors of our approximation.

We can roughly estimate the physical length scale  $\ell_D$  on which perturbations will be damped as  $\ell_D \simeq 2\pi/p_D$ , where  $p_D(t)$  is determined from (28) by setting

$$\int^{t_{nr}} \Gamma(t) dt = 1 \quad (35)$$

and using the relativistic expression for  $\eta$  (33). We neglect heat conduction since  $A \simeq 0$  in the relativistic limit, and perturbations are principally damped by viscosity.  $t_{nr}$  denotes the moment when neutrinos become non-relativistic and occurs when  $T \simeq m_\nu/3 \simeq 10$  eV. The physical lengthscale on which perturbations are damped is

$$\ell_D \simeq 2\pi \sqrt{\frac{\tau t_{nr}}{15}} . \quad (36)$$

This is approximately the distance one would estimate that a relativistic neutrino could travel via a random walk in time  $t$ : a neutrino can take  $N = t/\tau$  steps of length  $\tau$ , so on average each neutrino travels a distance  $\sqrt{N}\tau$ . One expects the neutrino number density to become homogeneous on scales shorter than  $\sim t/\sqrt{N}$ , which was the estimated damping scale of neutrino density perturbations given in [10]. To summarize, if one includes interactions for a hot dark matter candidate, perturbations are damped by diffusion rather than by free-streaming. The length scale on which perturbations are washed out is therefore shorter, and the power spectrum should look more like that of cold dark matter, as claimed in [10].

We have neglected mass effects in (36), because our fluid parameters are only correct to within factors of 3 or 4. To properly determine the damping scale, we would need accurate

determinations of  $\chi$ ,  $\eta$  and  $\zeta$ , valid as the temperature drops past the neutrino mass. We could then integrate  $\Gamma$  over all times to determine  $\ell_D$ . Instead, to check that our relativistic approximation is acceptable, we have calculated  $\chi$  and  $\eta$  in the non-relativistic limit, matched them to the relativistic versions at  $T = m_\nu$ , and calculated  $\ell_D$ . The damping scale is (to within a factor of 2) the same as in (36).

The comoving scale up to which density perturbations are washed out by neutrino diffusion is approximately

$$\lambda_D = \frac{2\pi}{k_D} = \frac{T_{nr}}{T_0} \ell_D \simeq 3 \sqrt{\frac{\tau}{\text{Mpc}}} \text{ Mpc} \quad (37)$$

where  $\tau$  is the co-moving mean free path at  $T_{nr} = 10$  eV. As discussed in section 2, the neutrino mean free path is bounded below by the arrival of the neutrinos from SN1987A. This means that the scale on which perturbations are damped is also bounded below, and we need to check that galaxy-sized perturbations do indeed survive.

The supernova bound implies that the timescale for neutrino self-interactions at  $t_{nr}$  (when  $T_\nu = 10$  eV) must satisfy

$$\tau_{nr} \gtrsim \frac{n_0}{n_{nr}} \frac{\sigma_{sn}}{\sigma_{nr}} \tau_{sn} = \left( \frac{T_0}{T_{nr}} \right)^3 \left( \frac{s_{sn}}{s_{nr}} \right)^{\pm 1} \tau_{sn} \quad (38)$$

where a 0 subscript means the present value of a given magnitude,  $n$  is the neutrino number density,  $\sigma$  is the cross section, and  $\tau_{sn}$  is the distance to the LMC.  $s_{nr}$  and  $s_{sn}$  are respectively the centre-of-mass energy for the neutrino self-interaction at  $T_\nu = 10$  eV, and for scattering a supernova neutrino off the neutrino CMB. The ratio  $s_{sn}/s_{nr}$  is to the power +1 if the cross section scales as  $s$  (exchanged boson mass  $> \sqrt{s_{sn}}$ ), and to the power  $-1$  for a cross section scaling as  $1/s$  (exchanged boson mass  $< \sqrt{s_{nr}}$ ). For a non-renormalisable cross section, the supernova bound implies that perturbations on comoving scales  $\lesssim 10^{-2}$  Mpc are damped by neutrino diffusion (or  $\sim 10^{-7}$  Mpc for a cross section that scales as  $1/s$ ). The supernova bound on the cross section is therefore consistent with the survival of galaxy scale perturbations, whether the neutrino stickiness is due to the exchange of light or heavy particles.

### 3.3 Evolution in the non-relativistic regime and final power spectrum.

We have seen that the sticky neutrinos behave quite differently from standard neutrinos while they are relativistic. However, once they are non-relativistic, they become a pressureless dust to lowest order, like most other dark matter candidates. If the neutrino cross section scales as  $s$ , then the interactions become weaker as the temperature drops, and our fluid approximation breaks down. However, if the neutrino interaction timescale is of order the horizon size then the interaction is irrelevant. On the other hand, if our fluid approximation is still valid, then we know that the viscosity and heat conduction coefficients are proportional to the pressure [25], and the pressure is negligible. So we expect the non-relativistic, interacting neutrinos to behave like standard neutrinos, if the interaction is short range. A longer range interaction (exchanged boson mass  $\ll m_\nu$ ) is potentially more complicated, particularly if the interactions are still strong today. Dark matter with long-range interactions has been considered in [13]; we neglect it here. Constraints on dark matter self-interactions from galaxy formation and evolution in the non-linear regime have been estimated in [12]. The authors argue that the average interaction length of dark matter today must be longer than 100 Mpc. Sticky neutrinos who interact

via the exchange of a  $m \sim \text{MeV}$  boson and who are consistent with the supernova bound are essentially non-interacting today (the interaction rate scales as  $a^{-5}$  or  $a^{-7}$ ), so they easily satisfy the constraints listed in [12].

Based on our analytic results, we expect the power spectrum of interacting HDM to be similar to standard CDM on large and intermediate scales, but damped on small scales by neutrino diffusion. We are working in the fluid approximation, so to compute the power spectrum, we must numerically integrate the Einstein equations and the energy conservation equation (14) for an imperfect fluid. This is considerably simpler than integrating the coupled Einstein and Boltzmann equations.

Let us briefly outline here the equations and approximations used. We perform our calculations in the synchronous gauge using the metric (3). The equations governing the evolution of perturbations are more easily written in Fourier space. If we only consider only scalar-type perturbations, the metric perturbations can be written in terms of two scalar fields  $h(k, t)$  and  $\eta(k, t)$  as follows [27]:

$$h_{ij}(x, t) = \int d^3k e^{i\mathbf{k}\mathbf{x}} [k_i k_j h(k, t) + \{k_i k_j - \frac{1}{3} \delta_{ij} 6\eta(k, t)\}] . \quad (39)$$

As in the FRW case, the equations governing the evolution of matter density perturbations can be derived from the conservation equation (14). For a relativistic fluid, with viscosity but no bulk viscosity or heat conduction, equations (25) and (26) become:

$$\dot{\delta} = -\frac{4}{3} \left( \theta - \frac{1}{2} \dot{h} \right) \quad (40)$$

$$\dot{\theta} + \frac{\dot{a}}{a} \theta = p^2 \left( \frac{\delta}{4} - \frac{\eta}{\rho} (\theta - 3\dot{\eta}) \right) \quad (41)$$

This results differs from [7] or [27] because our matter does not behave as a perfect fluid. As the temperature drops past the neutrino mass, the neutrinos no longer behave as relativistic particles. The evolution of density perturbations in the  $E \sim m$  regime is complicated by the time-dependent relation between the neutrino momentum and energy [27]. This adds terms to (40) and (41) and makes the numerical calculation of the pressure and density more difficult [32]. To simplify the calculation, we assume the transition from a relativistic gas to pressureless dust to be instantaneous. We also assume that neutrino interactions are frozen in the non-relativistic regime due to the small mean velocity. As a result, we treat the neutrino gas as CDM particles with zero velocity in the synchronous gauge, whose evolution is given by:

$$\dot{\delta} = \frac{1}{2} \dot{h} \quad (42)$$

As previously noted, perturbations in a relativistic fluid are damped by viscosity. The off-diagonal spatial elements of the stress-energy tensor (parametrized by the viscosity) are also gauge-invariant, which makes them simple to calculate. We therefore include only the viscosity (no heat conduction coefficient and no bulk viscosity) in our numerical calculation of the power spectrum. We take the physical interaction time  $\tau$  to scale as  $a^5$  (comoving  $\tau \sim a^4$ ), and assume an initial Harrison–Zel’dovich spectrum of perturbations from inflation.

In Fig. 1 we give the numerical integration of the previous set of differential equations coupled with the equations for the evolution of metric perturbations, baryons and photons. We used a modified version of the COSMIC package made publicly available by Berstchinger and Ma, and

described in [27]. The dashed line corresponds to standard CDM and is plotted for comparison. The three solid lines correspond, in decreasing amplitude, to zero viscosity (zero mean free path) and comoving mean free paths at  $T_\gamma = 10$  eV of  $10^{-2}$  Mpc and 0.1 Mpc. In the case of zero viscosity, the neutrino perturbations oscillate as acoustic waves around the CDM power spectrum. The behaviour is similar to the baryon-photon plasma where the interactions cause the perturbations in the baryon component to oscillate as sound waves. When the viscosity is introduced the oscillations are damped approximately at the scale given by the diffusion length. The effect of viscosity is to damp the power spectrum at the scale given by (37): 3 and 7 Mpc $^{-1}$  respectively for co-moving mean free paths of 0.01 and 0.1 Mpc. The damping is a purely fluid effect and it has nothing to do with gravitation as was already remarked in [16].

One should notice that the overall power spectrum amplitude is not completely accurate on scales  $k \simeq 0.3 \text{Mpc}^{-1}$ . We assumed, when calculating the power spectrum, that the neutrino gas underwent an instantaneous transition from radiation to pressureless dust at  $T_\gamma = 10$  eV. For the purpose of showing the effect of damping on small scales due to viscosity, the instantaneous transition approximation is quite accurate, and simplifies the calculations.

To summarize, neutrino viscosity helps to reduce power on small scales while keeping most of the features of CDM. In this way, it alleviates the problems that plague the standard CDM model, such as large scale pairwise velocity dispersion, that come from having too little power on large ( $\geq 30 h^{-1}$  Mpc) scales relative to small ( $\leq 10 h^{-1}$  Mpc) ones. A detailed analysis of the success of our model will require N-body simulations to compare with observations as performed, for example, by [9]. However, such analysis goes beyond the scope of the present paper.

## 4 Phase Space Constraints

The main argument against neutrinos as DM candidates was put forward by Gunn and Tremaine [33]. They argued that the large phase space density of galaxies was close to the density of a degenerate Fermi gas and they virtually ruled out neutrinos as dark matter candidates. The argument has been refined since [34, 35]. Currently, the tightest constraints come from two dwarf spheroidal galaxies within our local group: Draco and Ursa Minor. In those galaxies, if the spatial distribution of dark matter and luminous stars is similar, the central dark matter densities needed to explain the kinematic data are  $\rho_{DM} = 0.5 - 1 M_\odot \text{pc}^3$ . For non-interacting neutrinos, the conservation of the phase space density implies [36]:

$$m_\nu \geq 170 \text{eV} \left( \frac{1 \text{kpc}}{r_{DM}} \right)^{3/4} \left( \frac{0.1 M_\odot \text{pc}^{-3}}{\rho_{DM}} \right)^{1/8} g_\nu^{-1/4} \quad (43)$$

To overcome this limit and bring  $m_\nu$  down to 30 eV, the core radius of the DM,  $r_{DM}$ , should be almost two orders of magnitude larger than the one of the luminous stars, making the mass to light ratio of the system close to  $10^6 M_\odot / L_\odot$ . For sticky neutrinos the bound is very similar [10].

However, the halo of dwarf galaxies is not only a difficult test for fermionic DM, but any dissipationless galaxy formation scenario. The density of an object that formed without dissipation uniquely specifies its redshift of formation [37]:

$$1 + z_{turn} \simeq 4 h_{50}^{-2/3} \left( \frac{\rho_D}{10^{-2} M_\odot \text{pc}^{-3}} \right) \quad (44)$$

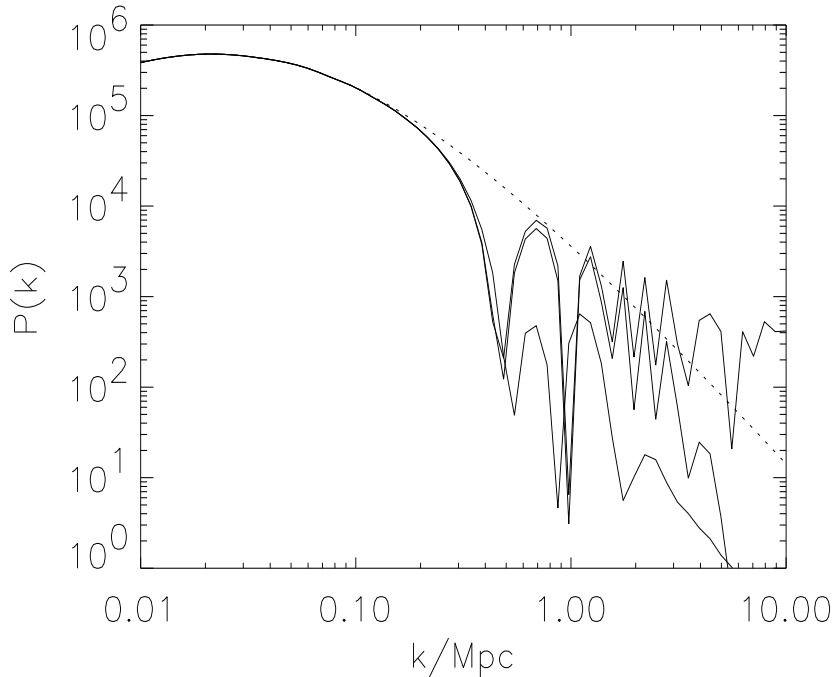


Figure 1: Power spectrum for three different “sticky neutrino” models. The solid lines correspond (in decreasing amplitude) to interactions with no neutrino diffusion, and with co-moving mean free paths at  $T_\gamma = 10$  eV of .01 Mpc and .1 Mpc. These give rise to estimated damping scales of  $7 \text{ Mpc}^{-1}$  and  $3 \text{ Mpc}^{-1}$ , respectively. The dashed line corresponds to standard CDM and is plotted for comparison. The y-axis scale is arbitrary. We took  $H_o = 50 \text{ Km s}^{-1} \text{ Mpc}^{-1}$ .

where  $\rho_D$  is the central density of the system. For Draco and Ursa Minor, the previous equation shows that  $(1 + z_{\text{turn}}) \simeq 30$ . In CDM, MDM and similar scenarios [38] the first structures turn around at about  $(1 + z_{\text{turn}}) \simeq 10$  and to bring down the previous limit would be necessary to have a DM core radius 4 times larger than the luminous matter and  $M/L \simeq 1000 M_\odot/L_\odot$  for these dwarfs, which is also rather high. Lacking a clear understanding of star and galaxy formation one can not disregard the possibility that a significant fraction of the halo is baryonic or dissipational [39], which would weaken the limit on the neutrino mass.

## 5 Discussion

As mentioned in the introduction, the “standard” CDM model, when normalized to the amplitude of temperature anisotropies measured by COBE/DMR, has too much power on small scales. Several modifications were proposed that agree better with observations, such as, for instance, hot plus cold dark matter models, CDM models with cosmological constant, tilted power spectrum and variants thereof. However, if one is going to add free parameters to one’s dark matter model, it is in principle as reasonable to add interactions for a single component as to mix two (or three) components. As an example of this, we have considered a dark matter model consist-

ing of  $\sim 30$  eV neutrino with a large  $\nu\nu$  scattering cross section. We have argued in this paper that these interacting neutrinos, although not easy to produce in a reasonable particle physics model, may be a viable dark matter candidate. Unlike ordinary hot dark matter, they do not free-stream. In section 3 we estimated the damping scale associated with neutrino diffusion and calculated the power spectrum. The current upper limits on the  $\nu\nu$  cross section permit a damping scale shorter than  $1Mpc$  and accordingly galaxy scale density perturbations will survive. Stronger upper limits on the cross section will increase the damping scale and could invalidate the model. The main positive feature of our model is that the power on small scales is reduced by neutrino diffusion while on larger scales we retain the main features of standard CDM. The neutrino–neutrino cross section, which is the only free parameter, could possibly be tuned to give “the right amount of power on small scales”. The main handicap is that interacting neutrinos, like ordinary neutrinos, can not constitute the halos of dwarf galaxies. However, it can be argued that dwarfs are difficult to explain in any dark matter scenario, so this may not be a fatal problem.

## acknowledgements

We would like to thank Georg Raffelt for useful conversations, and F. A.-B. is grateful to the Sonderforschungsbereich 375-95 for support when this work was completed.

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